Operating Temperature and Heat Capacity of a Light Bulb Filament – An Experimental Problem used in the German Physics Olympiad

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Abstract
We present an experimental problem used in the German national competition for the International Physics Olympiad, which, due to its emphasis on accurate data acquisition and rather unusual ways of evaluating the data, seems very suitable for testing the experimental abilities and creativity of the students.
The first part of the problem deals with the resistance of a light bulb filament as a function of temperature. The determination of the filament's temperature utilizes features of its black body radiation curve and the wavelength-dependent sensitivity of a photo diode. The analysis with the given instruments requires clever application of a logarithmic plot.
In the second task the heat capacity of the light bulb filament is investigated. This can be done by periodic heating and cooling of the filament. The process is driven by an appropriate voltage signal from a function generator and monitored using an oscilloscope. This task can also be used as a single problem, when the result of the first part is given.

Context

The selection competition for the German IPhO-team, the German Physics Olympiad, is organized by the Leibniz Institute for Science and Mathematics Education (IPN) at the University of Kiel and consists of four stages all carried out at a national level. In the first two stages the students solve sets of given problems at home. In each of the final two rounds the best participants come together and face two theoretical as well as two experimental exams, each lasting three to four hours.
The presented experimental problem is in parts based on an idea described by Kraftmakher (Kraftmakher 2004) and was posed for an exam in the fourth and final round of the competition in 2011. The 15 participating students had four hours time to work on it.

Preparation of the students

The second task involves an analogue oscilloscope and a function generator. To ensure that the participants could focus on physics rather than pure handling of the devices, an introduction on the equipment used was given two days before the exam.

For a hands-on training the group of students was split into pairs. One student was asked to set the function generator to given parameters, the other one then asked to find out these parameters by analyzing the signal with the oscilloscope. The roles were changed afterwards.
Special attention was paid to aspects that were considered potentially important for the problem. Some signals were similar to those that could have been used for the second task.

Problem

Introduction

The predominant process for heat transfer at high temperatures is by radiation. In this experiment you are asked to examine a filament of a light bulb at high temperatures.

Material

- Opaque box with four connectors, containing:
  - a light bulb with a nominal voltage of 6.0 V (maximum voltage 8.0 V)
  - a photo diode (Osram Components BPW 34)
- 9 V battery with a resistor of 100 Ω connected in series
- Power supply (variable DC voltage)
- 3 multimeters
- Function generator
- Oscilloscope
- Connectors and cables
- (10 0.1) Ω resistor
- 4 transparencies with graphs (see problem text and Figure 3)
- Graph paper, ruler, marker, pencil, eraser, adhesive tape

The uncertainty of the multimeters can be estimated as 2 % of the set measuring range. A list of their inner resistances is available. The ground contacts of oscilloscope and function generator have the same potential. Avoid short circuits!

Properties and use of the photo diode

The opaque box with the light bulb holds a photo diode. Using this diode, the light intensity of the bulb can be measured. When a constant voltage is applied to the diode in reverse direction, it acts as a current source with an output current proportional to the power of the incident radiation.

However, the constant of proportionality between incident power and output current depends on the wavelength of the light. For the diode in the box, this relative sensitivity is shown in Figure 1 (the data was taken from the datasheet mentioned in the references).

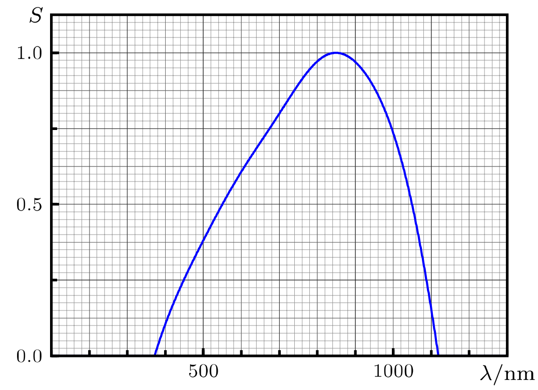

Figure 1: Relative sensitivity of the photo diode as function of the wavelength of the incident light.

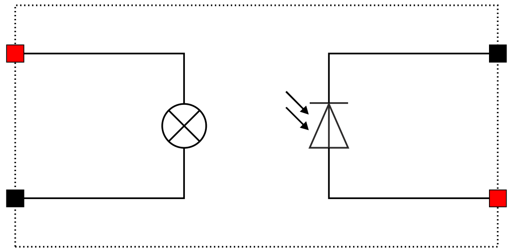
Figure 2: Illustration of the inner box layout as printed on the box given to the students.

Task 1

The graphs on the given transparencies (see Figure 3) show the current of such a photo diode when it is exposed to the radiation of a black body with different temperatures T, with no other sources of radiation present and, except for T all conditions left constant. The graphs differ only in the ways their axes are scaled. Note that the current signal is given in arbitrary units.

a) Give a brief and qualitative description, how the graphs on the transparencies can be calculated using the graph from Figure 1.

b) Examine the intensity of the light bulb for different voltages applied to it, in order to determine the electric resistance of the light bulb at different temperatures of the filament.
Plot the electric resistance R_B of the bulb against its temperature T_B and compare your result to a linear dependence of $\mathrm{R_B}$ on T_B.

c) Determine the temperature T_N of the light bulb filament at nominal voltage.

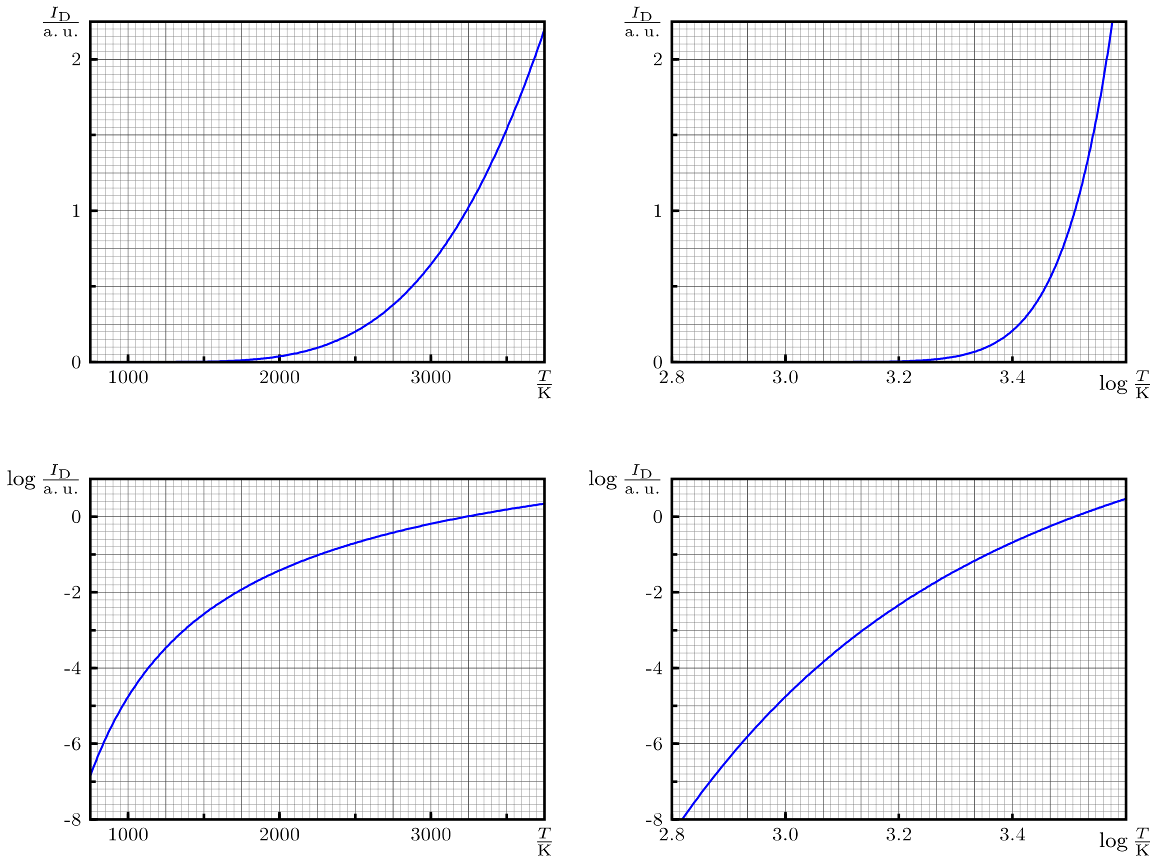

Figure 3: The plots of $I_D(T)$ with different scales as given on the transparencies for the students. Each plot for the participants was on an A4 transparency.

Task 2

The heat capacity of the filament can be determined with the given devices. To this end the light bulb can, for instance, be operated by square waves of low frequency produced by the function generator. The oscilloscope can be used to observe voltage curves.
Determine the heat capacity C of the filament. Name all approximations made, and the conditions for your result to be valid.

Solution to the experimental problem

Task 1

The fact that the photo diode's sensitivity depends on the wavelength of the incident radiation is crucial for this problem. Although the radiation power is proportional to T^4 according to Stefan-Boltzmann law, the sensitivity's dependence on the wavelength yields a different relation for the diode signal.
The output current I_D of the photo diode is proportional to the power of the incident light, weighted by the diode's spectral sensitivity $S(\lambda)$:

$$I_D \propto \int_0^\infty \frac{dE}{d\lambda}(T) \cdot S(\lambda) \cdot d\lambda$$

The spectral distribution $dE/d\lambda$ of the filament's radiation power is given by Planck's law. This integral can be evaluated numerically, which yields the given graphs. The current can only be given in arbitrary units, since the factor of proportionality is not known in this case.

As stated in the problem text, radiation is the dominant way of losing heat for the filament. Other means of heat transport can be neglected. This is also true for the absorption of radiation from the environment.
When the light bulb is operated at a constant voltage U_B with an according current I_B, a thermal equilibrium is quickly established. Thus the bulb's temperature can be derived from its power consumption, except for an unknown factor, here called α:

$$\sqrt[4]{I_B \cdot U_B} = \alpha \cdot T_B$$

In addition to the values of U_B and I_B, the current signal I_D of the photo diode can be measured for each data point.
The expected temperature dependence of I_D, given in the graphs of Figure 3, was discussed above. But since the theory only yields a proportionality, there is another unknown factor between the measured current and the numerical value given in the graph, here noted as β.
So instead of I_D over T_B as given in the graphs, the measurements only yield $\beta \cdot I_D$ over $\alpha \cdot T_B$. However, when plotting $\log(\beta \cdot I_D)$ over $\log(\alpha \cdot T_B)$ one obtains the same shape of the curve, while the coefficients α and β are merely reflected as shifts in x - and y - direction.
Therefore one can match the given logarithmic curve and the measured data by placing the transparency on top of a graph of the measured values. The same scales have to be used in both plots and the transparency may not be rotated, of course. Figure 4 shows the result with the data points shifted onto the given graph from Figure 3. The error bars are left out for reasons of clarity, which is justified since the errors become very small at higher temperatures and higher diode currents.
The value of α can be derived from the shift in x - direction and subsequently be used to determine the temperature T_B of the filament from its electrical power consumption in thermal equilibrium. The uncertainty of α can be estimated by variation of the shift. The data aligns well with the full-sized plots for a shift of about ± 2 mm, which is equivalent to a relative uncertainty of about 1.5 % for α. For the nominal voltage U_N one obtains a filament temperature of about $T_N = (2800 \pm 70)K$.
The plot of the bulb's resistance R_B as a function of T_B in Figure 5 shows very good linearity, with the resistance of the bulb approximately given by $R_B(T_B) = 22.9\frac{m\Omega}{K} \cdot T_B - 6.64\Omega$. Note that the linearity could be checked even without knowledge of α. The obtained relation $R_B(T_B)$ will prove to be useful in the next task.

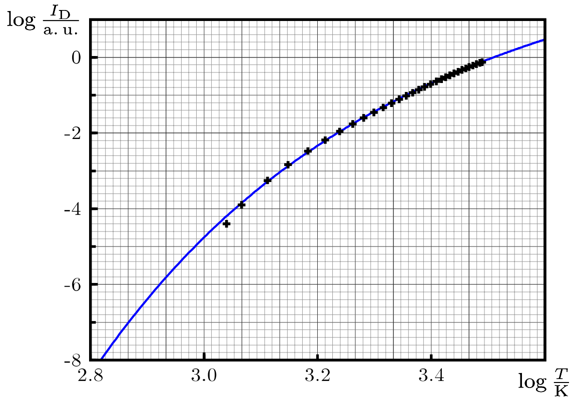

Figure 4: Experimental data points shifted onto the logarithmic plot of Figure 3.

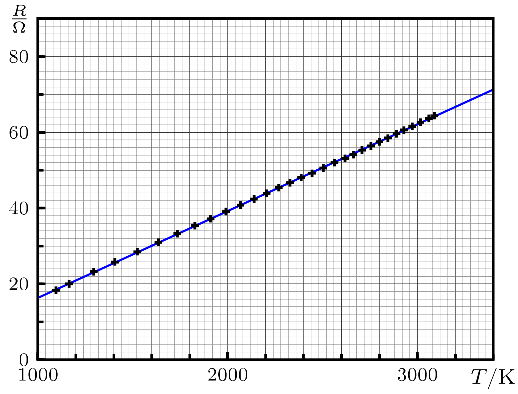

Figure 5: The results for $R_B(T)$ and a fitted linear function.

Task 2

The focus for the students' solutions for this task was on creative application of the given devices to determine the heat capacity, rather than obtaining a high accuracy. However, the students were asked to be aware of any assumptions or simplifications of their method, and to name them.

The heat capacity is relevant for processes out of thermal equilibrium, such as heating of the filament after switch-on, or cooling after switch-off. Since heating and cooling in the relevant high temperature range happens on rather short time scales (typically between 10 and 100 ms), the oscilloscope has to be used to study these processes. As the given oscilloscope does not provide a storage function, the observed processes are required to run periodically.

There are different ways to determine the heat capacity. We will present two of them here.

As suggested in the problem text, one can periodically switch the light bulb on and off using the function generator. This can be achieved with a square wave of amplitude u' and an offset of the same value. This means that the bulb is switched on with a voltage of $2 \cdot u'$ for the first part of the cycle and switched off the other time. The duration and relation of these periods can be changed with different frequencies and duty cycles of the signal.

To examine the light bulb, its voltage $U_B(t)$ and current $I_B(t)$ need to be measured. The voltage $U_0(t)$ supplied by the signal generator can directly be displayed on the oscilloscope. To measure the current, the given resistor can be connected in series to the bulb, with the voltage $U_R(t)$ over the resistor shown on the oscilloscope.

With this setup, the bulb's voltage and current can easily be derived from the observed quantities $U_0(t)$ and $U_R(t)$ using the following relations:

$$U_0(t) = U_B(t) + U_R(t)$$

$$U_R(t) = R \cdot I_R(t) = R \cdot I_B(t)$$

Note that the electric power $I_B(t) \cdot U_B(t)$ can not be used to determine the filament's temperature $T_B(t)$ here, as no thermal equilibrium is established. However, inverting $R_B(T_B)$ obtained in the first task, the resistance $R_B(t)$ can be used for that purpose.

One way to obtain the heat capacity is to directly determine the heat absorbed by the filament during a certain time interval of the heating period:

$$q = \int_{t1}^{t2} (I_B(t) \cdot U_B(t) - \alpha^4 \cdot T_B(t)^4) dt$$

The heat capacity then is:

$$C = \frac{q}{T_B(t_2) - T_B(t_1)}$$

This method requires careful and detailed recording of the curves to obtain good results.

A different and more practical approach is to consider the cooling intervals when the light bulb is switched off. One can determine the temperatures $T_B(t_1)$ at the end of a heating period and $T_B(t_2)$ at the beginning of the next one. In between the filament cools due to radiation:

$$C \cdot \dot{T}_B(t) = -\alpha^4 \cdot T_B(t)^4$$

After integration this yields:

$$\frac{1}{3 \cdot T_B(t_2)^3} - \frac{1}{3 \cdot T_B(t_1)^3} = \frac{\alpha^4}{C} \cdot (t_2 - t_1)$$

With different amplitudes, duty cycles and frequencies of the supply voltage, the initial temperature and the duration of the cooling period can be altered. The formula above suggests a plot of the left hand side expression as a function of the cooling time $t_2 - t_1$ as shown in Figure 6.

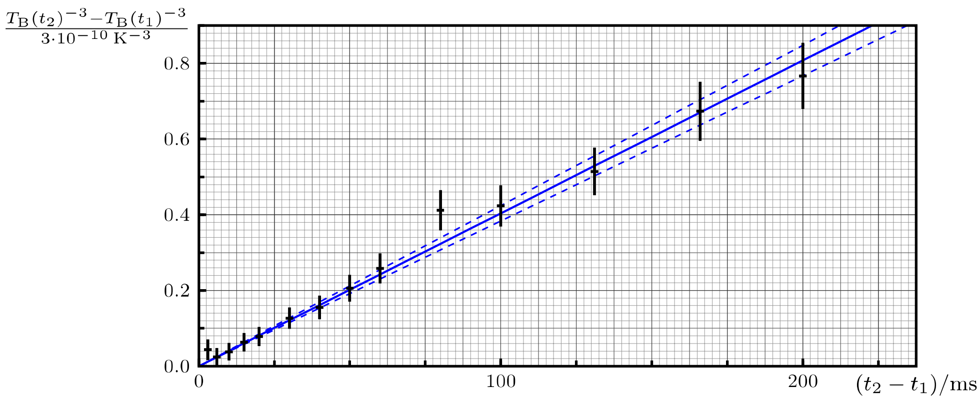

Figure 6: Plot to determine the heat capacity from the cooling phase of the filament. The dashed lines represent a 5 % variation of the fitted curve's slope.

Different measured quantities and previous results are used to calculate T_B. Accordingly the uncertainty of the plotted left hand side expression shown in Figure 6 is determined by many factors. However, the main contribution results from the two voltage readings on the analogue oscilloscope. The relative uncertainty of the time difference on the abscissa is negligible.

With the tools available during the exam, the uncertainty of the slope can only be estimated. A relative uncertainty of 5 %, as indicated in Figure 6, appears to be justified. For the calculation of C, the uncertainty of α discussed before also has to be considered.

Using the value of α determined before and the slope of the linear fit yields the heat capacity with $C = (2.2 \pm 0.2) \cdot 10^{-5} \frac{J}{K}$.

In addition to the uncertainty of the result it is also important to recognize the assumptions used in the presented techniques to determine C. For the two methods, the heat capacity was assumed to be constant in the examined temperature intervals. This is an approximation, as the specific heat capacity of the material may vary in the rather large temperature range. The specific heat capacity of tungsten for example increases by about 20 % in the temperature range from 1500 K to 2500 K according to (White 1997). Additionally the possibility of other parts than the filament changing temperature (thus contributing to the heat capacity) was neglected.

Within the scope of this exam, the students could not be expected to test these assumptions, but should be aware of them.

Conclusions

The two experimental tasks, albeit considered very interesting by the students, proved to be too extensive for the given amount of time. Even though roughly two thirds of the 15 candidates were able to achieve reasonable results in the first tasks only two of them received close to full marks. The second task was more or less successfully solved by only one of the students. These results can to some extent be attributed to the lack of time but may also be an effect of the unusual format of the experimental problem.

Altogether it can be said that the students certainly profited from the intensive training on the equipment used and that they were rather motivated to work on the problem. For the purpose of differentiating between the students in the competition the problem could have been shortened considerably, e.g. to only one of the two tasks.

Acknowledgements

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