Integrating Aspects of Geography in Physics Teaching

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Abstract
The underlying physical concepts and principles of the phenomena discussed in high-school geography texts concerning physics are vaguely stated and explanations are superficial. In addition, there is no quantitative treatment, which would be essential for a deeper understanding. By way of three examples, this paper illustrates the application of the principles studied in physics to the quantitative treatment of geographic phenomena: The height of a tide is estimated, and then the same technique is used to estimate two other heights.

1. Introduction

Hungarian high-school curricula contain two years of geography. Physical geography is taught in the first year, preceding most of, or, in some schools all physics instruction. Since the background knowledge in physics is lacking, students learn a significant part of their geography without really understanding it.

Later on, as students receive the background knowledge in physics, it becomes possible to revisit geographic phenomena. The ability to establish and apply quantities represents a higher level of understanding than conceptual knowledge, even though the modelling of a complicated phenomenon to be approached within the limits of high-school mathematics involves a great deal of simplification. Complexity of the treatment may vary in a wide spectrum, depending on the problem and on the extent to which the questions are structured. The following example will be heavily mathematical at high-school level. However, with all the necessary guidance given, it works well with advanced students (18 year-olds) who can appreciate the power of mathematics.

2. What would be the height of a tide on an Earth without continents?

The choice of tides was motivated by the observation that the treatment of tides in most geography books as well as a vast majority of popular science web pages (e.g. Figure 1) is based on the same misconception: the two lunar tidal bulges have two different reasons: the bulge facing the Moon is due to the attraction of the Moon while the one on the far side is due to the centrifugal force acting on the water rotating with the Earth around the centre of mass of the Earth-Moon system.

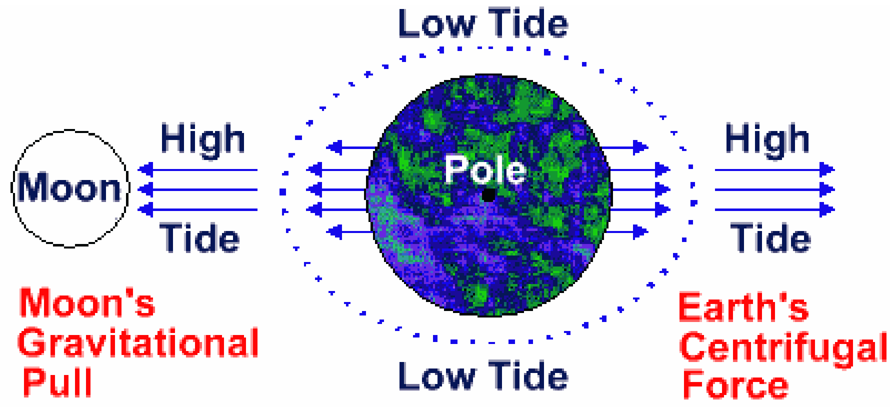

Figure 1.

Typical illustration. (To make it worse, this diagram gives the wrong impressions that the Moon orbits in the equatorial plane and that the centrifugal force is exerted by the Earth.) http://www.boatsafe.com/kids/earthtides.gif

This explanation is not only wrong because tides are caused by the non-uniform nature of the gravitational field, but it also involves a total confusion about the centrifugal force. Unfortunately, two hours of physics per week are not enough to introduce non-inertial reference frames. We do emphasize, however, in solving physics problems, that what happens is not a consequence of one force or another: all forces acting on an object should be considered. We also emphasize that the conclusion will not depend on the choice of the reference frame. The centrifugal force in itself cannot be the reason for the bulge since it only exists if a rotating frame is used. It is all right to use a rotating frame but the conclusion that there are two tidal bulges has to be the same in an inertial frame, too. To provide a better explanation, let us investigate the approximate range of water levels caused by the non-uniform gravitation of the Moon. (Naturally, textbook illustrations must exaggerate the size of the tidal bulges. But what is their true height?) For the sake of simplicity, assume a featureless Earth that consists of a rigid sphere surrounded by an ocean that will readily assume the equipotential shape. For further simplicity, only the effect of the Moon is considered, the effect of the Sun is disregarded.

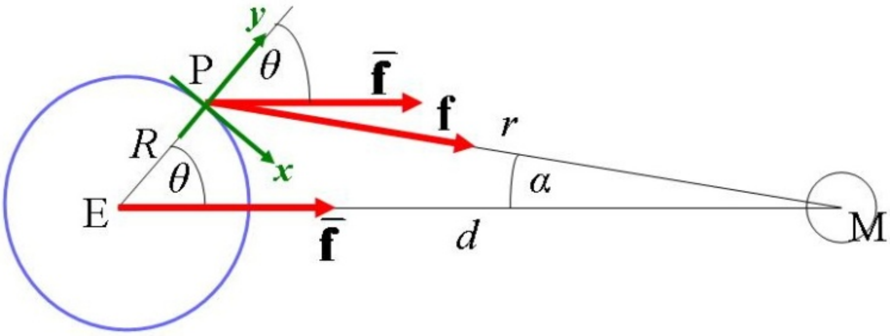

Fig. 2

Figure 2 shows the Earth, the Moon and the notations that will be used. Since $d \approx 60R$. (R is much smaller than d). **f** denotes the gravitational force per unit mass exerted by the Moon, at a particular location an angle θ away from the Earth-Moon axis.

The actual force per unit mass **f** can be considered the sum of $\bar{\mathbf{f}}$ acting at the centre (that is the same everywhere and does not cause deformation) and a force $\mathbf{f} - \bar{\mathbf{f}}$ that is often called the tide generating force. The deformation of the ocean surface is only caused by $\mathbf{f} - \bar{\mathbf{f}}$ and that result is independent of the reference frame. Since any stretching or compressing force will be counteracted by forces arising within the water, we need to concentrate on the tangential (x) component of the tide generating force: if equilibrium is to be maintained, it is that component that needs to be compensated for by a sea surface sloping down in the opposite direction.

$$f_x - \bar{f}_x = \frac{GM_M}{r^2}\sin(\theta+\alpha) - \frac{GM_M}{d^2}\sin\theta, \quad \text{where} \quad \sin(\theta+\alpha) = \frac{d}{r}\sin\theta$$

according to the sine rule for ΔEMP. By substituting and factorizing, an expression containing the difference of the inverse cubes of r and d is obtained:

$$f_x - \bar{f}_x = \frac{GM_M}{r^2}\cdot\frac{d}{r}\sin\theta - \frac{GM_M}{d^2}\sin\theta = GM_M d\left(\frac{1}{r^3} - \frac{1}{d^3}\right)\sin\theta$$

Using algebraic identities and the fact that r is about the same as d, this difference can be approximated as follows:

$$d\left(\frac{1}{r^3} - \frac{1}{d^3}\right) = \frac{d^3 - r^3}{d^2 r^3} = \frac{(d-r)(d^2 + rd + r^2)}{d^2 r^3} \approx \frac{(d-r)\cdot 3d^2}{d^5} = \frac{3(d-r)}{d^3} =$$

$$= \frac{3(d^2 - r^2)}{d^3(d+r)} \approx \frac{3(d^2 - r^2)}{d^3 \cdot 2d} = \frac{3}{d^2}\cdot\frac{d^2 - r^2}{2d^2}, \text{ Hence}$$

$$f_x - \bar{f}_x = \frac{3GM_M}{d^2}\cdot\sin\theta\cdot\frac{d^2 - r^2}{2d^2}$$

Now we have the difference of the squares in the numerator, so the cosine rule

($d^2 - r^2 = 2Rd\cos\theta - R^2$) can be applied and the distance *r* is thus eliminated:

$$f_x - \overline{f}_x = \frac{3GM_M}{d^2} \cdot \sin\theta \cdot \frac{2Rd\cos\theta - R^2}{2d^2},$$

$$f_x - \overline{f}_x = \frac{3GM_M}{d^2} \cdot \sin\theta \cdot \left(\frac{R}{d}\cos\theta - \frac{1}{2}\left(\frac{R}{d}\right)^2 \right).$$

Finally, the tangential force per unit mass is expressed as a multiple of the gravitational acceleration *g*. (Any variation of *g* with position is ignored since this is only an estimation.)

Since $g = \frac{GM_E}{R^2}$, we have $f_x - \overline{f}_x = \frac{3}{2} \cdot \frac{M_M}{M_E} \cdot \left(\frac{R}{d}\right)^3 \cdot \left(\sin 2\theta - \frac{R}{d}\sin\theta \right) \cdot g$

Students are familiar with the idea of not making a distinction between the sine and tangent of an angle that is small. Since the angle of inclination of the sea surface is very small, the quantity multiplying *g* in the above expression can be considered the slope of the sea surface: For the surface to remain in equilibrium, this tangential force component needs to be balanced by a downhill component of the Earth's gravity on the slope (Figure 3).

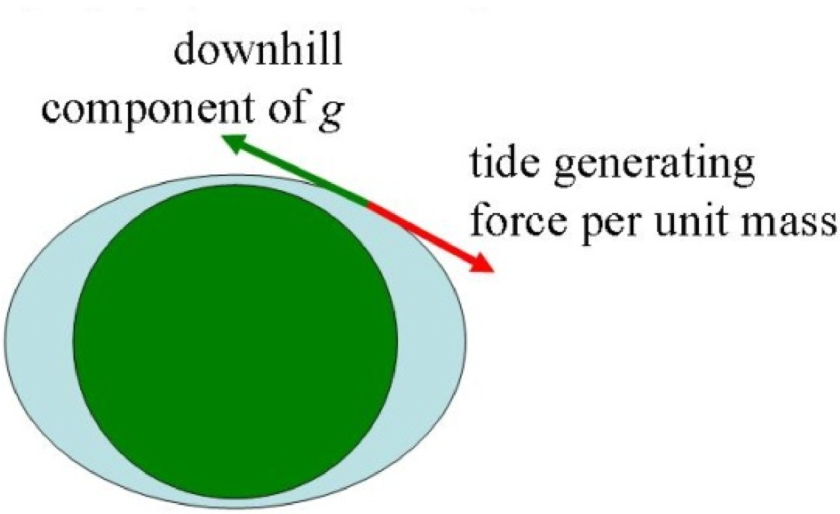

Figure 3.

In the expression of the slope, the second term is only significant next to the first term when θ is close to 90°, but in that case, too, its value is very small because of R << d.

Thus the maximum slope is found at $\theta \approx \pi/4$, where $\sin 2\theta = 1$. Since $\frac{M_M}{M_E} = 0.0123 \approx \frac{1}{81}$ and $\frac{R}{d} = 0.0166 \approx \frac{1}{60}$, the maximum slope is estimated to be

$$\frac{3}{2}\cdot\frac{M_M}{M_E}\cdot\left(\frac{R}{d}\right)^3 \approx 8.4\times10^{-8},$$

which means a rise of 8.4 cm over a distance of 1000 km. Slope means rise over unit distance. The total rise from low tide to high tide on our featureless ocean-covered Earth is obtained by adding up these rises for the total distance between a low and a high. That means values of θ between $\pi/2$ and 0, that is, over a distance of a quadrant circle, $R\pi/2$. Therefore the difference between the highest and lowest water levels is obtained by calculating the area under the curve in Figure 4:

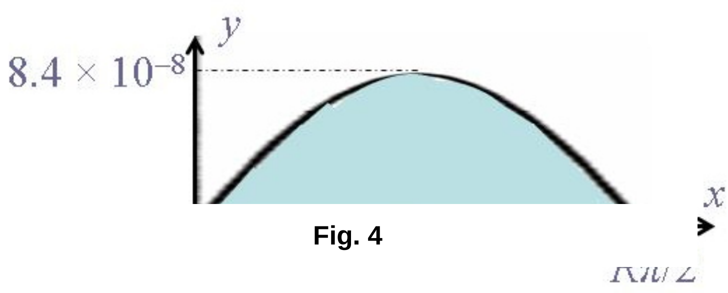

Fig. 4

$$y = 8.4\times10^{-8}\cdot\sin 2\theta = 8.4\times10^{-8}\cdot\sin\frac{2x}{R}$$

on the interval of $x = 0$ to $x = \frac{R\pi}{2}$.

(The second term does not need to be considered since a small deviation from the sine 2θ curve at its tail where the values are low does not influence a rough estimation.)

If students do not know calculus, it is enough to tell them that the area under $y = \sin x$ from 0 to π is 2 to get the area in question:

$$2\cdot\frac{R}{2}\cdot 8.4\times10^{-8} = 8.4\times10^{-8}\cdot 6370\times10^{3}\,\text{m} = \mathbf{0.54\,m},$$

which is not a very large height, but it is still an immense amount of water sloshing around as the Earth is turning underneath. It is important to note that the meaning of the word “tide” in everyday speech is not the rise and fall of a hypothetical continent-free ocean, but the rise and fall of water levels at a particular coastal location, which may be much larger than the result obtained, and which depends on many things, such as bulges reflected from continents (resonant motions of sea basins), shoreline topography (sea floor slope, estuaries and bays), position with respect to the Moon’s orbit, and the effect of the Sun.

3. How flat is the Earth?

Let us move on to another effect that determines the shape of the surface. Geography texts all mention the flattened shape of the Earth, but again, the distinction between ellipsoid, geoid and local topography is not made clear.

The following estimation aims to determine the order of magnitude of the difference of polar and equatorial radii of the Earth-ellipsoid by only using the mean radius and the rotation period. For simplicity, it assumes an Earth that is covered in a thick layer of water, thicker than in reality, that will readily assume the ellipsoidal shape owing to rotation. A rotating reference frame attached to the Earth is used. This time, it is the tangential component of the centrifugal force (Figure 5) that needs to be compensated for by an appropriate sloping of the surface. With a calculation completely analogous to that carried out with tides, the tangential component is written as a multiple of *g*, with the coefficient being a function of latitude *φ.*

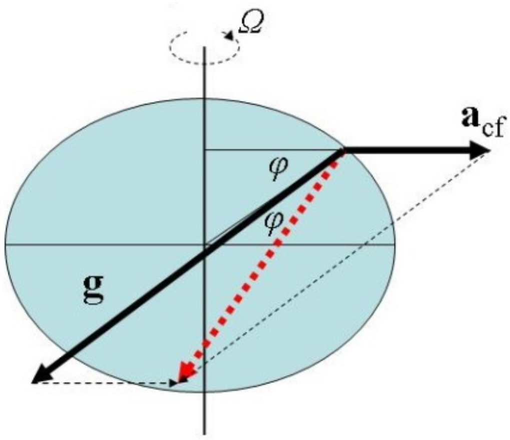

Figure 5.

The angular speed of the Earth is:

$$\Omega = \frac{2\pi}{24 \cdot 3600} = 7.3 \times 10^{-5}\ \mathrm{s}^{-1}.$$

The tangential component of the centrifugal acceleration at a latitude *φ* is

$$a_{cf,x} = R\cos\varphi \cdot \Omega^2 \cdot \sin\varphi = \frac{R}{2}\Omega^2 \cdot \sin 2\varphi$$

Since this is only a rough estimation, we can calculate with the average radius of 6370 km.

With this radius

$$\frac{R}{2}\Omega^2 = 0.017 = 1.7 \times 10^{-3} g$$

(This maximum occurs at about 45° of latitude.) At a latitude *φ*, this requires a sea slope of 1.7×10^{-3} sin2*φ* for equilibrium. The total rise from pole to Equator is the area under the curve

$y = 1.7 \times 10^{-3} \sin\frac{2x}{R}$ over the interval $x = 0$ to $x = \frac{R\pi}{2}$,

which is $2 \cdot \frac{R}{2} \cdot 1.7 \times 10^{-3} = 1.7 \times 10^{-3} \cdot 6370 \times 10^3 \mathrm{m} = \mathbf{11\,km}$.

The actual values of polar and equatorial radii are 6357 km and 6378 km, their difference is 21 km. So the estimation gives the correct order of magnitude, and we cannot expect more since it was based on an over-simplification of the situation. Note that the 21 km difference is much greater than the height variations owing to tides. Naturally, this oblate spheroidal shape serves as a baseline to which tidal heights should be added. It is also instructive to compare this difference of the order of 10 km to the deviation of the geoid (the true equipotential surface) from the ellipsoid. The map of Figure 6 shows this deviation, which does not exceed 110 m anywhere.

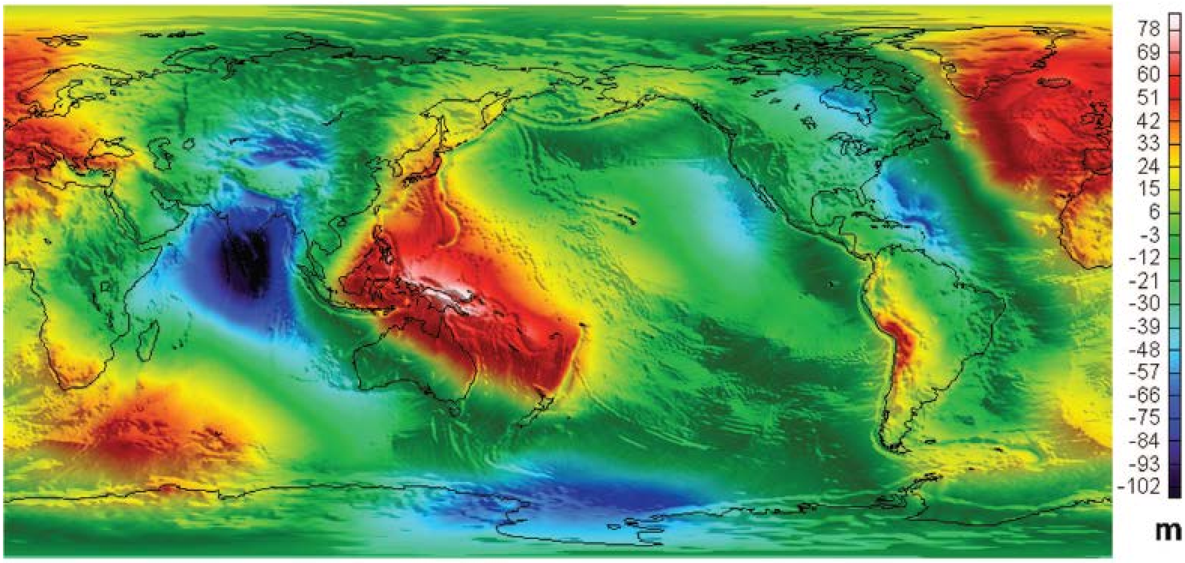

Figure 6. Deviations from the ellipsoid. [3]

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4. What is the rise of the ocean surface across the Gulf Stream?

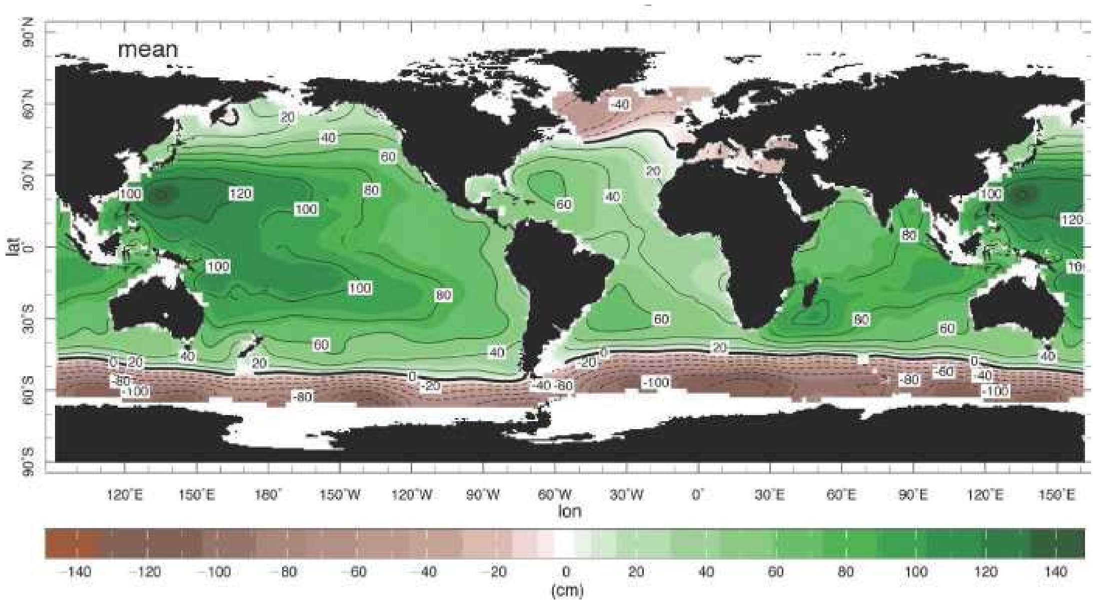


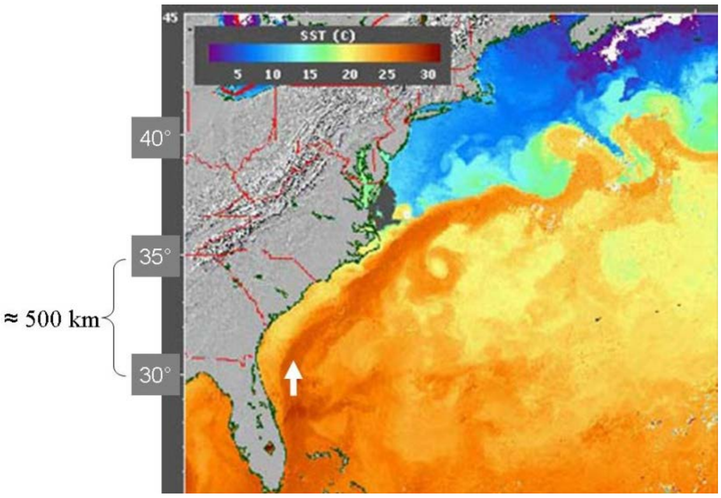
Figure 7. Deviations of mean sea surface from marine geoid. [4]

Figure 7 shows the deviations of the mean ocean surface from the marine geoid: all within the range of plus 1.5 metres to minus 1.5 metres. We can observe the regions where strong and steady ocean currents flow, for example next to the east coasts of continents. These currents are discussed by all geography texts, the Gulf Stream being the best known of them since it also features a lot in predictions and guesses about climate change that receive a lot of public attention. The Gulf Stream can be observed as a dark orange band in the false-colour image of Figure 8 where colours indicate surface temperature. Since one degree on the globe corresponds to about 110 km, we can estimate the width of the Gulf Stream to be 100 km. It does not have sharp boundaries, so this will pass as an order-of-magnitude estimation. Geography texts also mention that the Coriolis force plays a role in the formation of these currents, although again, without a thorough explanation (most of them do not even say that it depends on speed). In addition to the missing explanation, students may be interested to hear that ocean currents also involve a sloping of the sea surface, and this slope is easy to estimate. In the estimation, we will concentrate on the region marked with the white arrow in Figure 8, where the Gulf stream flows northwards at N30° latitude.

The horizontal Coriolis acceleration is

$\mathbf{a}_{Cor} = 2\mathbf{v} \times \boldsymbol{\omega}$

where $\boldsymbol{\omega}$ is the local vertical component of the Earth's angular velocity ($\omega = \Omega \cdot \sin\varphi$) as shown in Figure 9.



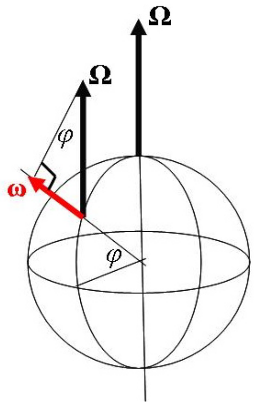


Figure 8 **Figure 9**

http://oceanmotion.org/images/gatheringdata/modern-gulf-sst.jpg

If a parcel of water at 30° latitude is travelling northwards on the surface at uniform speed *v*, its Coriolis acceleration is eastwards as shown in Figure 10.

$$a_{Cor} = 2\omega \cdot v = 2\Omega \sin\varphi \cdot v \, .$$

To keep the equilibrium, the sea surface is sloping up towards the east and the slope is obtained if the Coriolis acceleration is divided by *g*.

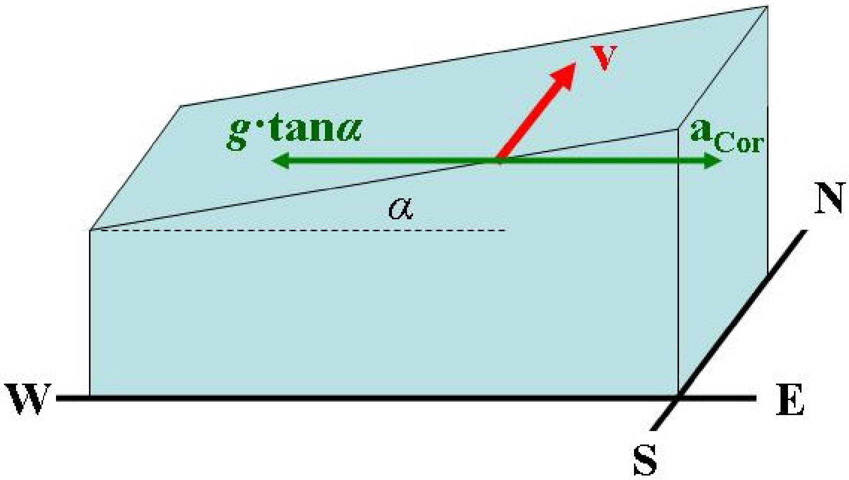


Figure 10

With *v* in the order of 1 m/s substituted, the slope of the surface needs to be

$$\frac{2\Omega \sin\varphi \cdot v}{g} = \frac{2 \cdot 7.3\times 10^{-5} \cdot \sin 30° \cdot 1}{10} = 7.3\times 10^{-6} \approx 10^{-5}$$

that is, about 1 metre over a distance of 100 km, which is the approximate width of the stream. The order-of-magnitude result agrees with reality. (Note that this is a sloping of the mean sea surface, since even ocean waves produce larger variations in height.)

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6. Conclusion

These were a few examples of quantitative problems constructed to support understanding geography. The explanation given to the student necessarily involves a simplification. Even though a model may only describe a very restricted reality if its scope and limitations are made clear, it is always possible to elaborate the model later on when we have more background knowledge at hand. However, it is impossible to take that further step ahead from the hazy explanations provided by geography texts. Phenomena addressed by geography texts that lend themselves to physics problems include winds, cyclonic rotations, air humidity, thermals, the forming of precipitation, water and earthquake waves, the slope angle of sand dunes, the earth-atmosphere energy budget and many others. In addition to a link between disciplines, the discussion of such phenomena also provides an opportunity to synthesize knowledge across different chapters of physics, such as mechanics and thermal physics.

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